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Investing for Retirement Income: A Comparison of Asset Allocations and Spending Strategies[†]

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Abstract

We study the performance of different investment and spending strategies for retirement. Investment strategies include wealth-focused glide paths that combine equities with short-term, high-quality fixed income. We also consider an income-focused glide path that combines a moderate equity allocation at retirement and an inflation-protected bond portfolio that uses liability-driven investing. Spending rules include fixed spending (similar to the 4% rule), flexible spending, as well as nominal and real annuitization. We examine simulated lifetimes with either stochastic longevity or fixed longevity of 30 years in retirement.

We find that, for all spending strategies, an income-focused asset allocation delivers similar retirement income to the wealth-focused allocations we study while offering better protection against market, interest rate, and inflation risk. We also find that a glide path with an LDI portfolio offers a better tradeoff between income growth and income risk management. Finally, our results suggest that high equity exposure in retirement is an inadequate tool to manage longevity risk.

1. Introduction

Sound retirement planning requires thinking not only about how to invest assets, but also how to spend them. Moreover, as emphasized by Merton (2014), retirement investors are exposed to risks beyond the volatility of their assets. Since retirees typically want to maintain a stable standard of living, they are exposed to both interest rate and inflation risk. Inflation has the potential to erode the purchasing power of an investor's nest egg and reduce the standard of living it can support in retirement. Similarly, lower interest rates may decrease the returns available on bonds and reduce the retirement income a given balance can generate.

This paper examines how different asset allocations and spending strategies can support stable retirement income. We assume a hypothetical investor who starts saving at 25, retires at 65, and eventually passes away. Longevity is either fixed or simulated, based on a mortality table. An economic environment that includes stock market returns, interest rates, and inflation is simulated for each period of the investor's life.

We consider three asset allocations. Each allocation is a *glide path*, which specifies the investor's portfolio at each age. Two wealth-focused glide paths combine equities and short-term, nominal bonds. Both glide paths gradually increase the allocation to fixed income as the investor approaches retirement. This approach is similar to that of target date funds, which seek to reduce the volatility of investors' assets near retirement.[1] The difference between the two wealth-focused allocations is their *landing point*, the proportion allocated to equities at retirement. We consider both high (50%) and moderate (25%) equity landing points.

[1] Target date funds are a popular choice for retirement investors, with $2.3T in assets in the US at the end of 2019 (Kephart et al., 2020).

We also consider an income-focused glide path, which seeks to reduce the volatility of retirement income, rather than the volatility of assets, as the investor approaches retirement. This objective is consistent with academic work in life cycle finance (e.g., Bodie et al., 1992; Viceira, 2001; Cocco et al., 2005). The income-focused glide path combines a moderate equity landing point of 25% with a portfolio of inflation-indexed bonds that matches the duration of an inflation-indexed retirement income stream. Such an allocation is designed to address market, inflation, and interest rate risk.

For each of the three asset allocations, we evaluate four spending strategies: fixed spending, similar to the 4% rule of Bengen (1994); flexible spending; a nominal annuity; and an inflation-indexed annuity. Under fixed spending, at age 65 the investor computes the annual income that her nest egg can provide for the next 30 years. She then withdraws the same amount every year, adjusted for inflation. Under flexible spending, the investor adjusts her annual spending each year based on her account balance and her conditional life expectancy. Since the investor uses current information to compute her spending in each period, she can achieve better outcomes than she would by basing her spending solely on the information she had at age 65. Indeed, flexible spending generates higher average income than fixed spending, at the cost of annual adjustments that make lifetime income less smooth.

Retirees are likely to incorporate elements of both fixed and flexible spending. Under fixed spending, consumption is perfectly smooth unless assets are depleted. Under flexible spending, assets are never depleted but consumption is subject to wide swings from year to year. Davis (2010) studies combinations of fixed and flexible spending rules, and highlights the benefits of allowing some flexibility in spending. The appropriate degree of flexibility will vary among retirees based on their preference for smooth retirement income. The composition of spending may also be a consideration. As noted by Lee (2013), discretionary spending can be substantial for more affluent retirees. Such retirees could absorb shocks to retirement income through adjustments in their discretionary spending on items such as travel or luxury goods. Conversely, retirees who allocate a high proportion of their income to essential spending (such as rent or health expenses) may prefer a more predictable income stream.

Annuities are an important alternative since they can help investors manage longevity risk. We compare nominal annuities to real annuities to see how inflation interacts with longevity risk. Although inflation-indexed annuities are not readily available for purchase, many public pension schemes feature cost-of-living adjustments that help offset inflation. Moreover, higher lifetime payments can often be obtained by deferring the first payment. As noted by Munnell et al. (2020), retirees in the US can perform a “Social Security bridge” by collecting Social Security as late as possible while funding their early retirement spending from other sources. This approach essentially allows one to purchase additional inflation-indexed, lifetime income.

Some annuities offer payments that increase by a fixed percentage (“COLA”, or cost-of-living, adjustment) every year. These annuities do not offer the same hedging properties as a real annuity since the yearly adjustment does not vary with realized inflation. However, they can emulate some of the benefits of a real annuity, such as more stable purchasing power in the long run: annuities with a COLA adjustment start with a lower initial payment than nominal annuities, but the payment maintains more of its purchasing power over time. Breakeven inflation rates can provide guidance when choosing a percentage for the COLA adjustment.

We assess the performance of each investment and spending rule combination by examining the distribution of retirement income across simulations. In particular, we focus on the average retirement standard of living achieved under each strategy, as well as the dispersion of outcomes and downside risk. Downside risk is an important consideration for retirement investors, who may not have flexibility around their retirement date. For example, working longer may not be practical for someone retiring early because of health issues or for an employee who got laid off a few years before her planned retirement date. These individual issues may also coincide with poor economic and market conditions: Chen et al. (2020) find that forced retirement is more likely during stock market downturns. In this case, having an asset allocation that can support retirement income under adverse circumstances is crucial.

Previous work (Chirputkar et al., 2019) has looked at the backtested performance of the S&P STRIDE family of indices, which measure the performance of an income-focused glide path. The sample period starts in 2003 because reliable data on inflation-indexed bonds (in this case, Treasury Inflation-Protected Securities, or TIPS) do not extend further back in time. We address this limitation by simulating economic environments. Our model, described in the appendix, is calibrated to economic scenarios that reflect US historical experience, with more emphasis on recent decades. In particular, we assume lower bond returns than the historical average to reflect current low yields, as lower interest rates can substantially affect the performance of different retirement income strategies. Additional tests confirm that our results are robust to different input values.

Our key findings are as follows.

- Average assets at age 65 are virtually equal for the wealth-focused allocation with a high equity landing point and the income-focused allocation. The moderate equity landing point of the income-focused glide path reduces the dispersion of outcomes significantly without reducing the average.
- For fixed spending, the income-focused allocation has the lowest failure rate. The strategy generates similar lifetime income to the high-equity wealth-focused allocation, and higher income than the moderate-equity wealth-focused allocation. The income-focused strategy also offers better downside protection, as measured by the 10th percentile of average income.
- For flexible spending, the income-focused allocation outperforms the moderate-equity wealth-focused allocation on all measures, despite having similar equity exposure. The wealth-focused allocation with a high equity landing point offers the highest average income at the cost of much higher volatility. The income-focused glide path has the

highest 10th percentile of lifetime income, and median lifetime income is competitive with the high-equity wealth-focused strategy.

- High equity exposure in retirement is an inadequate tool to manage longevity risk. A higher exposure to equity leads to higher failure rates under fixed spending, even when longevity is higher than expected; the benefits under flexible spending are also limited. Annuities, which are designed to address longevity risk, are a more appropriate tool. They can also generate higher average income because of mortality pooling, though they require the investor to give up control of her annual spending and assets.
- Even moderate increases in inflation and decreases in interest rates can substantially reduce the income generated by the two wealth-focused strategies. The income-focused glide path is protected against such events by design.

Section 2 describes the simulation setup. Section 3 shows the results of a single simulation. In Section 4, we summarize the results of all simulations to assess how the distribution of retirement income generated by each strategy varies with longevity and economic conditions. In Section 5, we look at how retirement income evolves over time under different strategies. Finally, Section 6 examines how financial shocks (poor stock market returns, high inflation, or low interest rates) that may occur early in retirement can affect spending over the entire decumulation phase.

2. Simulation Setup

Our simulations follow a hypothetical investor over her lifetime, which includes both the accumulation and decumulation periods. The hypothetical investor starts saving at 25 and retires at 65. At the start of each year of the accumulation phase, the investor makes a contributions of $12,500, adjusted for inflation, to a retirement account.[2] The account is invested according to one of the three asset allocations detailed below. At retirement, the investor can either annuitize her entire account balance or continue to invest her savings and gradually spend them. In the latter case, retirement spending can be a fixed amount (in real terms) or a flexible amount that varies with the account balance, real interest rates, and conditional life expectancy.

The investor is assumed to live until at least 65. During the decumulation phase, the investor can either die in each period, with probabilities sourced from latest mortality tables, or live for a fixed period of 30 years. We compare those different types of life trajectories across 100,000 simulated histories of economic conditions. Each simulated history consists of 95 years (from age 25 to a maximum age of 120). For each year of simulated history, we generate stock market returns, inflation, and yields on nominal and inflation-indexed bonds of different maturities according to the methodology presented in Appendix A. The simulated economic environment affects both investment performance and spending behavior.

[2] In untabulated results, we also consider contributions that start at $5,000 and rise to $20,000 by the end of the accumulation period. The relative performance of all asset allocations and spending strategies is very similar to the baseline results with constant contributions. Therefore, our conclusions are also relevant for workers who increase their contributions over time due to wage growth or increase in contribution rate. See De Santis and Lee (2013) for a discussion.

2.1 Asset Allocations

Investors have access to three asset classes: stocks, inflation-indexed bonds, and nominal bonds. Bond maturities range from one to 30 years. All bonds are zero-coupon and default-free. We consider two wealth-focused allocations and one income-focused allocation, illustrated in **Exhibits 1**, **2**, and **3**. All allocations are rebalanced at the beginning of each year. Both wealth-focused allocations combine equities and five-year nominal bonds. The allocation to stocks starts at 100% and gradually decreases toward a high (50%) or moderate (25%) landing point at age 65. The percentage allocated to equities stays constant afterwards. The 50% landing point is close to the average among target date funds, while 25% is at the low end of the distribution.[3] Many target date funds have a significant allocation to nominal, short-term, high-quality fixed income (Chirputkar et al., 2019), a feature that we represent with the allocation to five-year nominal bonds.

EXHIBIT 1

Wealth-focused Glide Path with High Equity Landing Point (WF-50%)

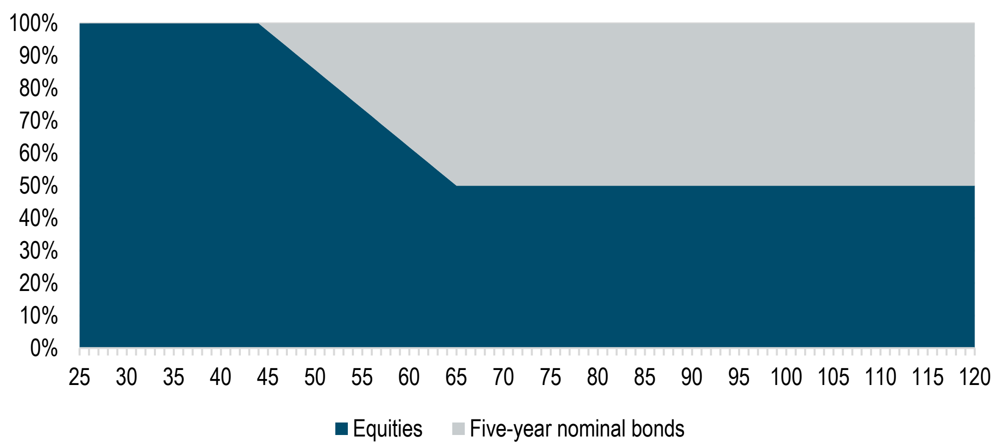
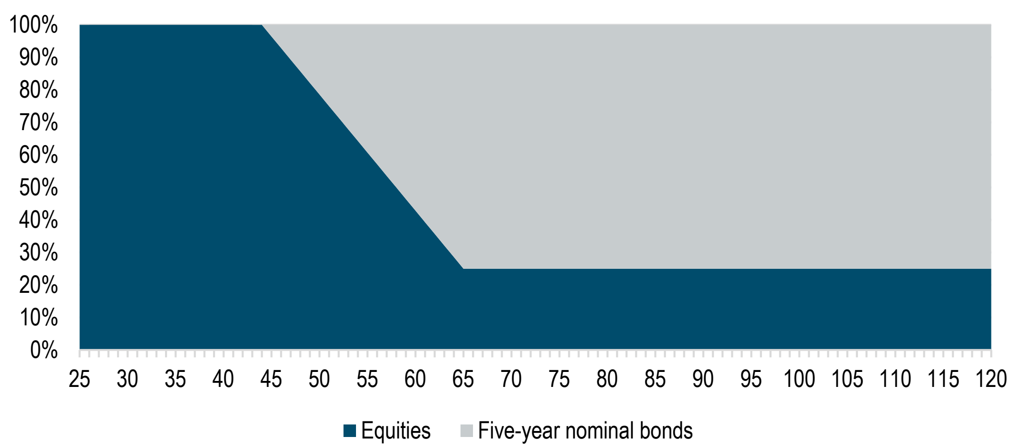


EXHIBIT 2

Wealth-focused Glide Path with Moderate Equity Landing Point (WF-25%)

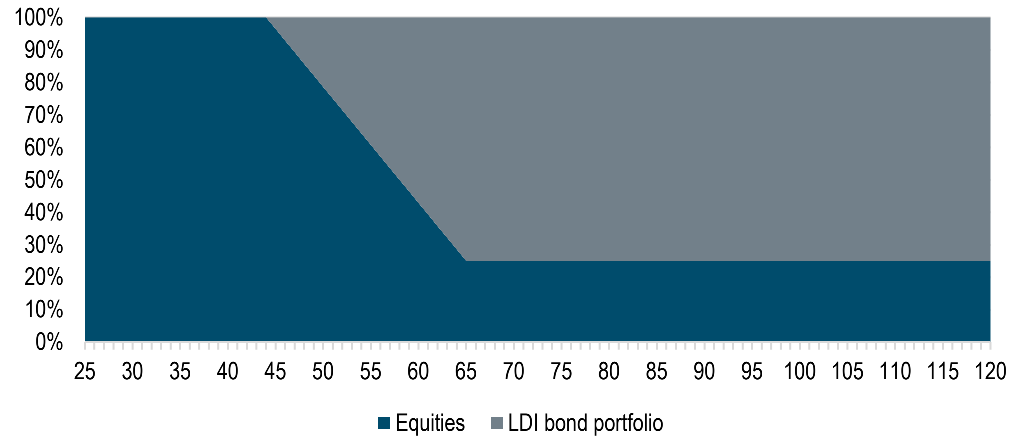


[3] Hamish Preston and Adrián Carranza Araujo, "S&P Target Date Scorecard" (white paper, S&P Dow Jones Indices, March 2021). Report 1 in the document shows the allocation to equities by vintage. Also note the small allocation to TIPS.

We also consider an income-focused glide path. The strategy has the same allocation to equities as the wealth-focused allocation with a moderate landing point. However, instead of investing in short-term nominal fixed income, it invests in a portfolio of inflation-indexed bonds that seek to match the duration of a stream of real income representing annual retirement spending.[4]

EXHIBIT 3

Income-focused Glide Path with Moderate Equity Landing Point (IF-25%)



The number of projected payments is based on the investor's conditional life expectancy, multiplied by 1.5 to provide a buffer against longevity risk. For example, at age 65, the investor's conditional life expectancy is around 20 years. With the longevity buffer, the number of projected annual payments becomes 30. Therefore, at age 65, the portfolio of inflation-indexed bonds seeks to match the duration of a stream of 30 equal inflation-adjusted payments. At age 75, the investor's conditional life expectancy is around 12 years. With the longevity buffer, the number of projected annual payments becomes 18. Thanks to the ongoing duration matching, the value of the bond portfolio closely matches the cost of the retirement liability, providing protection against interest rate risk in addition to inflation risk.

To distinguish between the three glide paths, we use the abbreviations WF-50%, WF-25%, and IF-25%. The percentage refers to the equity landing point at retirement. WF stands for wealth-focused, while IF stands for income-focused.

2.2 Spending Rules and Mortality

We study four spending rules: fixed spending, flexible spending, and annuitization with either nominal or inflation-indexed payments.

Both types of annuities are priced using mortality probabilities derived from Social Security Administration mortality tables and averaged across genders. The nominal annuity payments are discounted using the 10-year yield on nominal risk-free bonds at the end of the accumulation period,

[4] For a real-world example of an index that incorporates an LDI portfolio, see Mathieu Pellerin, "Income-Focused Strategy Indices Show Resilience in 2020 (Part 1)," S&P Indexology (blog), S&P Dow Jones Indices, February 16, 2021.

while the 10-year real yield is used for inflation-indexed annuities. The choice of a conservative discount rate (which results in higher annuity prices) seeks to make annuity pricing more realistic given that we ignore additional costs that might affect real-world annuity pricing.

Inflation-indexed annuities generate constant real income throughout retirement. Although inflation-indexed annuities are not readily available for purchase, they represent a useful comparison point to measure the impact of inflation on nominal annuities. Nominal annuities typically start out with a higher payment than real annuities, because the 10-year nominal yield is higher as it reflects inflation expectations. The payment then decreases with time because of inflation. Again, the "payment" here is measured in units of purchasing power rather than nominal dollars. Under both types of annuitization, all of the investor's assets get exchanged for lifetime payments; the investor leaves no bequest behind upon death.

Under fixed and flexible spending, the investor keeps her retirement assets and bases her spending on the cost of a hypothetical retirement liability. The liability consists of $1 inflation-indexed payments lasting for 1.5 times the investor's conditional life expectancy, paid in full at the beginning of each period, on the investor's birthday. At age 65, conditional life expectancy is 20 years, so the cost of the liability is the present value of 30 payments based on the current real yield curve.

Under fixed spending, the investor determines her spending at age 65 based on the above calculation and keeps it fixed (in real terms) throughout retirement; payments are adjusted for inflation each year. When the real yield curve is equal to its average, the present value of the 30 equal payments is $23.51, which corresponds to spending 4.25% (or 1÷23.51) of the initial balance. The spending rate will be higher if interest rates at retirement are higher, and vice versa. A key metric for fixed spending is the probability of failure, defined as the probability of depleting all assets before death.

With flexible spending, the investor repeats the same calculation at the beginning of each period. Therefore, her spending will be proportional to her account balance at the beginning of the period and vary with interest rates. Also, her conditional life expectancy changes with each year of survival. For example, at age 65, the investor can expect her last birthday to occur at age 84. However, conditional on reaching age 75, she can expect her last birthday to occur at age 87. At age 75, the investor will plan for (87 – 75) x 1.5 = 18 future payments rather than (84 – 75) x 1.5 = 13.5.[5]

It is impossible to run out of money under flexible spending since the investor always spends a fraction of her balance. Still, her income could get unreasonably low over time. A useful yardstick is the difference between initial spending and the minimal spending reached during retirement. This measure reflects both temporary decreases in spending (perhaps because of a temporary market drop) and situations in which spending declines steadily. The latter can occur when longer-than-expected longevity interacts with low investment returns.

Another useful measure for flexible spending is the standard deviation of changes in annual spending. This quantity captures how volatile retirement spending was in a given simulated history.

[5] Fractional payments get rounded down.

A high value means that many abrupt changes in spending occurred; a low value means that spending was relatively smooth around its trajectory. The measure is meant to complement the difference between initial and minimum spending by emphasizing year-to-year fluctuations rather than steady declines or increases.

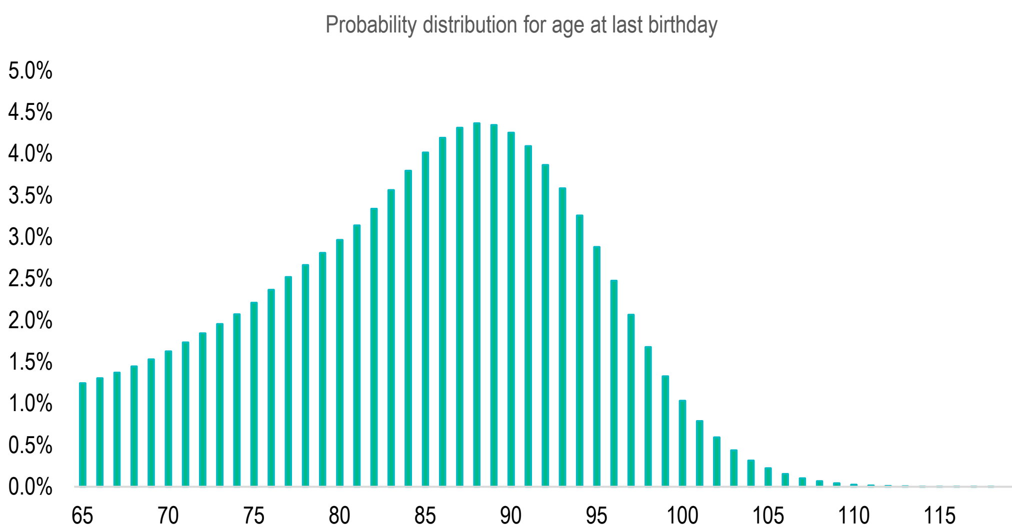
The LDI portfolio in the income-focused allocation seeks to hedge changes in the cost of the retirement liability under flexible spending. The LDI portfolio invests in the inflation-indexed bond that most closely matches the duration of the retirement liability. For small, parallel shifts in the real yield curve, the increase in value of the LDI portfolio will match the increase in the cost of the retirement liability.

Finally, **Exhibit 4** presents the distribution of mortality we use to price annuities and simulate longevity in our stochastic mortality results. The median retiree lives for 21 years. Less than 15% of retirees live to age 95, and less than 4% reach age 100. Retirees planning for 30 periods therefore have approximately a 15% chance of living that long, and results are largely driven by simulations in which the investor dies sooner. For this reason, we also present results conditional on living for 30 periods to focus on how the different strategies fare in a scenario with high longevity.

EXHIBIT 4

Mortality Probabilities Conditional on Living to Age 65

One-period mortality probabilities (q_x) are obtained from the SSA period mortality tables (see Endnote v) and averaged across genders. The probability of dying in the next year conditional on reaching age 65 + k is based on projected mortality in year 2020 + k to account for mortality improvement. P(Die at age x) = P(Survive to age x) – P(Survive to age x+1) and P(Survive to age x+1) = P(Survive to age x) * (1-q_x). The recursion starts with q_{65} and P(Survive to age 65) = 1.



Our main variable of interest is average lifetime retirement spending for a hypothetical retiree. This variable provides a measure of the standard of living achieved in retirement. We also look at the probability of running out of assets for fixed spending. For flexible spending, we report the gap between initial income and the minimal income reached during retirement, as well as the standard deviation of annual changes in spending.

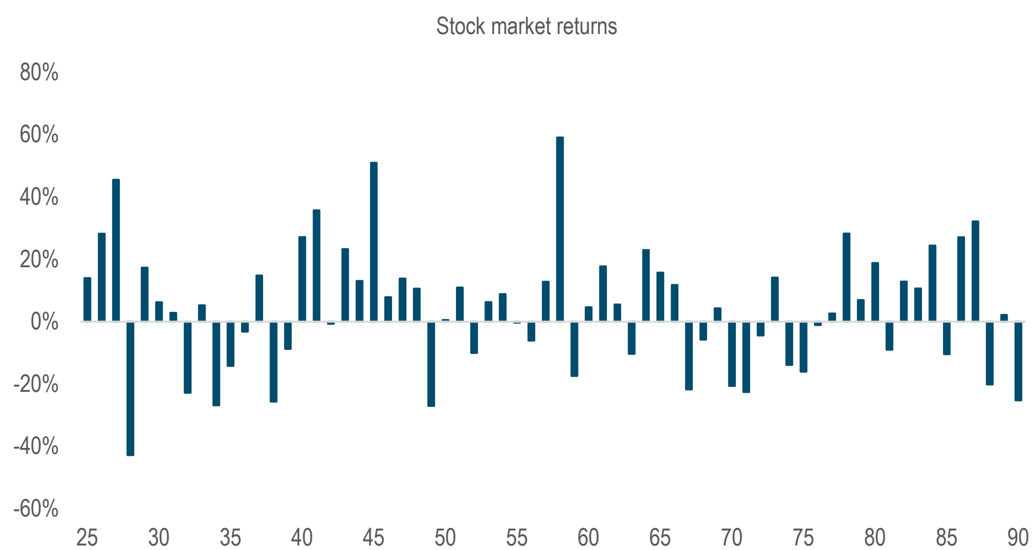
We compare 12 combinations of investing and spending strategies across the 100,000 simulated histories of stock returns, interest rates, and inflation. Unless noted otherwise, all units are in inflation-indexed, or real, dollars. Using real dollars facilitates the comparison of retirement spending at different points in time (for example, spending at age 65 vs. spending at age 80).

3. A Case Study

In this section, we show the output associated with a single simulation to provide intuition for our main results. In this simulation, the investor's last birthday happens at age 90. Therefore, she withdraws 26 payments before her death, starting on her 65th birthday. **Exhibit 5** shows the behavior of the stock market over the investor's lifetime, while **Exhibit 6** shows real interest rates and **Exhibit 7** shows inflation**.** Our hypothetical investor experienced typical stock market returns: the realized average of annual real returns over her life was 4.4%, close to the 5% expected real return in our setup. Real stock returns were 6.2% per year on average preretirement and 1.6% post-retirement. The realized volatility of annual returns was 19.8%, close to the 20% expected volatility.

EXHIBIT 5

Stock Market Returns Over the Investor's Life



Hypothetical performance is no guarantee of future results.

For illustrative purposes only. All simulations are based on a hypothetical probability distribution, not historical data. See Appendix A for details.

EXHIBIT 6

Real Yields Over the Investor's Life

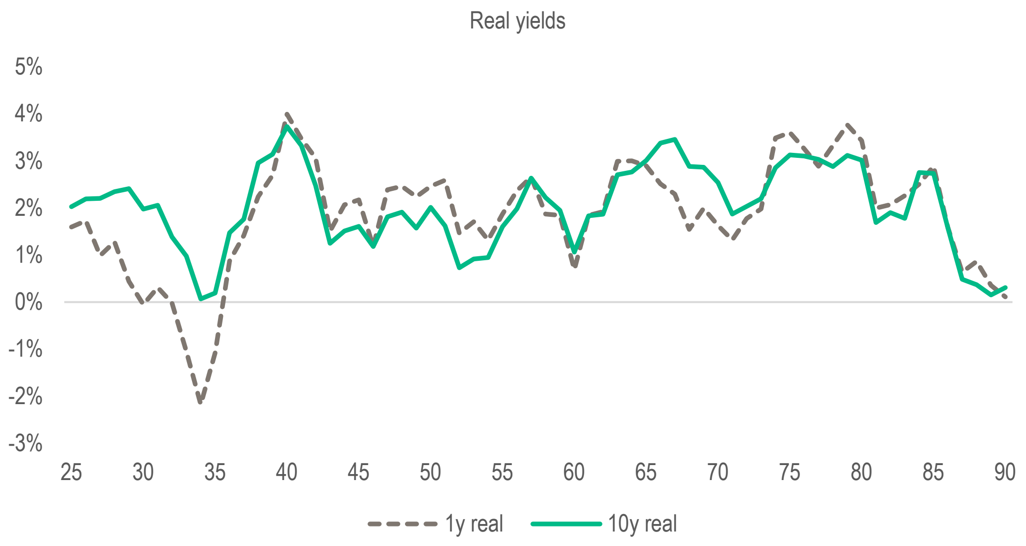


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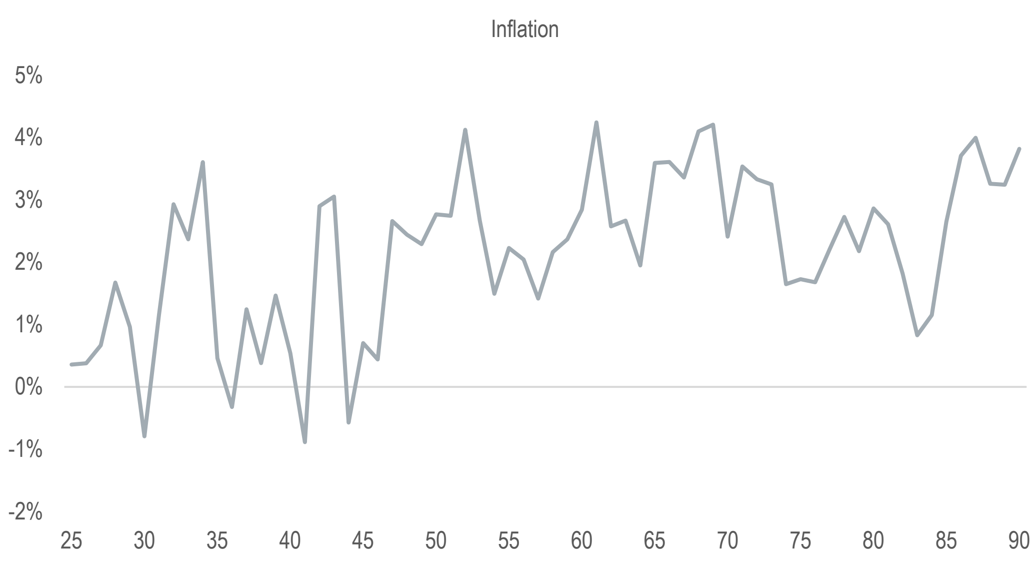
EXHIBIT 7

Inflation Over the Investor's Life



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Real yields fluctuated during the investor's lifetime, and short-term yields went negative early in the accumulation period. Inflation was typical, with a 2.2% realized average annual rate compared to a 2% expected value. Inflation was lower preretirement (1.7%) than post-retirement (2.8%). **Exhibit 8** shows that, under this environment, the 40 contributions of $12,500 would have grown to more than $1 million under both wealth-focused glide paths, WF-50% and WF-25%, as well as the income-focused glide path, IF-25%.

EXHIBIT 8

Initial Retirement Balance and Initial Income

WF-50% = 50% equity landing point and 5-year nominal bonds; WF-25% = 25% equity landing point and 5-year nominal bonds; IF-25% = 25% equity landing point and LDI portfolio. Initial assets at retirement are based on 40 annual contributions of $12,500 during the accumulation phase. All numbers are inflation-adjusted.

	WF-50%	WF-25%	IF-25%
Accumulated balance at age 65	1,388,164	1,242,561	1,183,627
Initial withdrawal (fixed and flexible spending)	66,545	59,565	56,740
Initial payment (nominal annuity)	109,720	98,212	93,554
Last payment (nominal annuity)	55,111	49,331	46,991
Payment (real annuity)	90,322	80,848	77,014

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The investor's initial spending rate for fixed and flexible spending is 4.8%. By comparison, the spending rate would be 4.25% when all real yields are equal to their assumed long-term averages. The higher spending rate occurs because, as seen from Exhibit 6, real yields are high prior to retirement. The 10-year rate at the beginning of retirement was 2.8%, compared to a long-run average of approximately 1.7%.

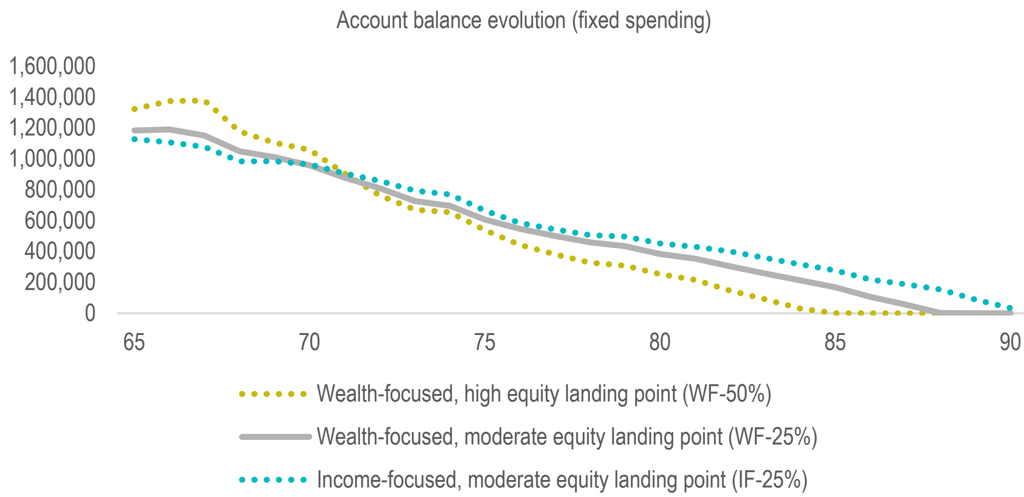
The investor has the option to annuitize her balance. If she opts for a nominal annuity, the first payment she receives is 7.9% of the accumulated balance. The payment rate of the real annuity is lower, at 6.5%. However, by the end of the investor's lifetime, the nominal annuity payment would have lost 50% of its purchasing power. This is true even though average inflation in retirement was 2.8%, higher than expected but still reasonably low. If the investor lives longer, the erosion in purchasing power would be even steeper.

If the investor retains her assets and opts for a fixed spending strategy, she spends the amount listed in Exhibit 8 as long as funds are available: $66,545 for WF-50%, $59,565 for WF-25%, and $56,740 for IF-25%. However, she would run out of money on her 85th birthday under WF-50% and on her 89th birthday under WF-25%. As can be seen in Panel A of **Exhibit 9**, poor stock market returns have a pronounced impact on WF-50%, which starts with the highest balance at retirement but then falls behind the other strategies. The income-focused allocation benefits from its moderate equity exposure and its LDI portfolio. The bonds in the LDI portfolio provide inflation protection and benefit from the high and decreasing interest rates in the decumulation period.

EXHIBIT 9

Investor's Balance at Beginning of Each Period

PANEL A: FIXED SPENDING



PANEL B: FLEXIBLE SPENDING

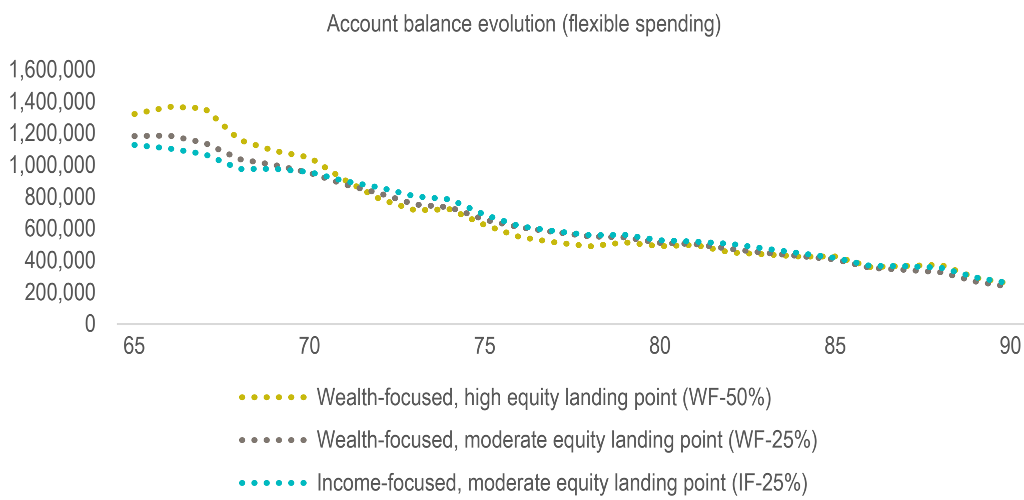


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EXHIBIT 10

Investor's Annual Spending for Flexible Spending Under the Three Asset Allocations

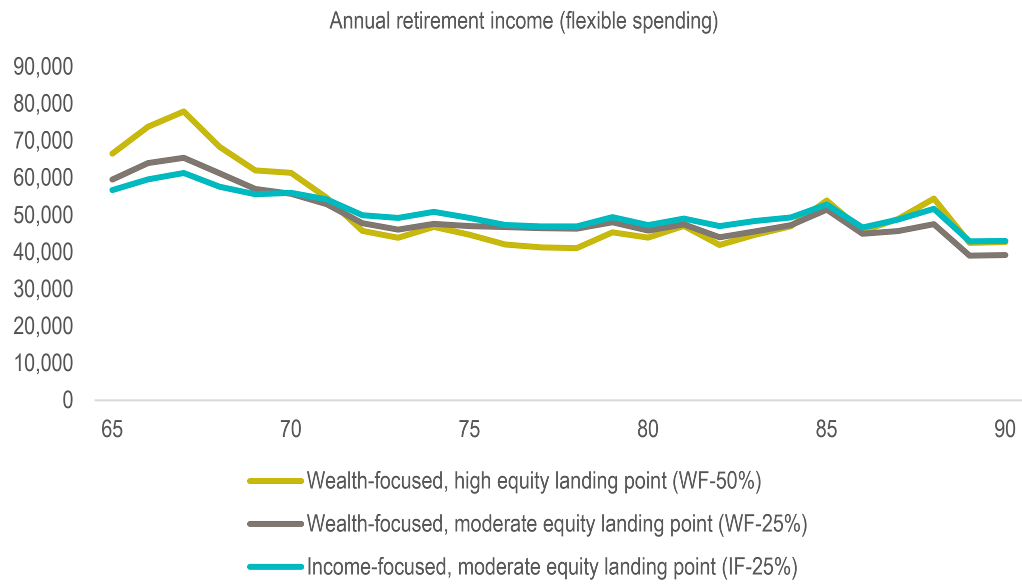

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Exhibit 10 shows the evolution of annual income under flexible spending. For all strategies, the investor reacts to the negative stock market returns around age 70 by cutting her spending. WF-25% starts with higher spending than IF-25% ($59,565 vs. $56,740) because of a higher initial balance. Nominal bonds outperform inflation-indexed bonds during the accumulation phase since inflation is lower than expected, leading to a higher balance for WF-25%. However, post-retirement inflation is higher than expected, and inflation erodes the returns of the nominal bond allocation until spending dips to $39,209 by age 90 for WF-25%, compared to $42,978 for IF-25%. Overall, the income-focused allocation yields the most stable spending.

Exhibit 11 shows the outcomes for this single simulation. In the next section, we will summarize and compare these outcomes across 100,000 simulations. In the current simulation, the investor makes 26 retirement withdrawals, which we can average to compare the standard of living in retirement. For example, with fixed spending under WF-50%, the first 20 withdrawals are $66,545, the 21st is $34,717 because the remaining balance is insufficient to cover a full withdrawal, and the five last withdrawals are $0. Average spending over retirement would be $52,524, and the simulation would count as a failure since the investor ran out of assets.

EXHIBIT 11

Outcomes for a Given Simulation

WF-50% = 50% equity landing point and 5-year nominal bonds; WF-25% = 25% equity landing point and 5-year nominal bonds; IF-25%= 25% equity landing point and LDI portfolio. Initial assets at retirement are based on 40 annual contributions of $12,500 during the accumulation phase. All numbers are inflation-adjusted.

	Fixed Spending			Flexible Spending		
	WF-50%	WF-25%	IF-25%	WF-50%	WF-25%	IF-25%
Average lifetime income	52,524	55,048	56,740	51,081	49,615	50,687
Failure	Y	Y	N	–	–	–
Min. - Init. income	–	–	–	–25,479	–20,532	–13,838
SD of annual income changes	–	–	–	5,390	3,214	2,993
Bequest	0	0	31,418	220,837	216,858	241,021

Hypothetical performance is no guarantee of future results.

For illustrative purposes only. All simulations are based on a hypothetical probability distribution, not historical data. See Appendix A for details.

On the flexible spending side, when comparing the minimum spending reached in retirement to its initial value, the income-focused strategy (IF-25%) shows the smallest reduction because of its lower allocation to stocks and the inflation indexing of its bond portfolio. Average income is lower under flexible than fixed spending for all strategies, but consumption over retirement is smoother since the investor does not run out of money, as seen from the bequest amounts.

The standard deviation of annual changes in spending shows that spending was less volatile under the income-focused strategy, confirming the intuition from Exhibit 10. The higher value for WF-50% reflects the additional volatility that comes with high equity exposure. Volatile equity returns cause sharp year-to-year changes in the accumulated balance, ultimately resulting in more volatile retirement spending.

4. Baseline Results

In this section, we focus on the standard of living achieved over the course of a complete retirement. The key measure we use is lifetime average retirement spending: for example, for a retiree who lives 20 years, we average spending at ages 65, 66, and so on, up to age 84. In addition, we look at the probability of running out of assets under fixed spending, and, for flexible spending, the variability of income during retirement.

Exhibit 12 presents the distribution of accumulated assets at retirement across 100,000 simulated financial histories based on each investment approach. The key takeaway is that initial retirement assets are similar across allocations. All strategies are fully invested in equities from ages 25 to 45. From 45 to 65, the glide paths gradually diverge as they reallocate from equities to fixed income until they reach their landing point. At age 55, halfway through the transition period, the wealth-focused allocation with a high equity landing point (WF-50%) has approximately 75% of assets invested in equities, while the other allocations (WF-25%, IF-25%) have around 60%.

WF-50% has the highest average initial assets, as expected from its higher equity exposure, though the difference is small. Median assets are virtually equal across strategies. By contrast, the equity exposure has a sizable effect on the range of outcomes. Despite leading to similar asset levels on average, strategies with more bond exposure have less dispersed outcomes, as seen from either the standard deviation or the range between the 10th and 90th percentile.

EXHIBIT 12

Balance at the Beginning of Retirement

WF-50% = 50% equity landing point and 5-year nominal bonds; WF-25% = 25% equity landing point and 5-year nominal bonds; IF-25% = 25% equity landing point and LDI portfolio. The average, standard deviation, and percentiles are taken across 100,000 simulated values of the initial balance. Initial assets at retirement are based on 40 annual contributions of $12,500 during the accumulation phase. All numbers are inflation-adjusted.

	WF-50%	WF-25%	IF-25%
Average	1,376,458	1,281,302	1,336,764
Standard deviation	1,160,925	958,502	1,015,512
10th percentile	478,436	510,097	524,258
50th percentile	1,051,940	1,021,311	1,060,300
90th percentile	2,590,589	2,314,909	2,422,626

Hypothetical performance is no guarantee of future results.

For illustrative purposes only. All simulations are based on a hypothetical probability distribution, not historical data. See Appendix A for details.

Exhibit 12 suggests that exposure to equity early in the glide path, which is identical across strategies, seems to be more important for the average outcome than exposure close to retirement. The equity landing point, however, matters for the variability of outcomes.

4.1 Fixed and Flexible Spending

Panel A of **Exhibit 13** presents the main results for fixed and flexible spending under stochastic mortality. For fixed spending, average retirement income is around $55,000 for all three investment strategies. The two strategies with a lower equity landing point have less-dispersed outcomes. For example, the standard deviation of lifetime average income is $49,273 for WF-50%, compared to $42,403 for the income-focused allocation, IF-25%.

Under fixed spending, the initial spending amount only depends on the account balance, the prevailing interest rates at age 65, and life expectancy at 65. Spending is then constant until death or until assets run out. High equity exposure in retirement increases the variability of the portfolio, thereby increasing both the probability of running out of assets and the probability of leaving behind a large bequest (because spending is never adjusted upward to consume "excess" assets). As a result, WF-50% has both the highest failure rate, 13.1%, and the highest average bequest, $985,197. IF-25% has the lowest failure rate of the three allocations, 8.5%, but also a lower average bequest, $699,023.

One way to measure downside risk is to look at the 10th percentile of lifetime average income. Panel A of Exhibit 13 shows that, under fixed spending, IF-25% generates the highest 10th percentile ($21,741), followed by WF-25% ($20,640), and WF-50% ($19,069). The income-focused strategy also generates the highest median income.

Under flexible spending, the investor can adjust her annual spending based on her assets and prevailing interest rates. For all allocations, this results in a higher average income and a lower average bequest than under fixed spending. The effect is especially marked for WF-50%, which has the highest percentage of equities in retirement. The volatility of retirement income also increases significantly (relative to fixed spending) for WF-50%.

The respective 10th and 50th percentiles of income are $20,726 and $49,068 for WF-50%, compared to $22,795 and $47,275 for IF-25%. For the difference between initial spending and the minimum reached during retirement, the income-focused allocation has the lowest drop on average, at –$10,590. The variability of annual changes in spending is twice as high for WF-50% compared to IF-25%. Under both fixed and flexible spending, Panel A of Exhibit 13 suggests that an income-focused allocation can help manage risk while delivering comparable outcomes to conventional strategies.

Panel B of Exhibit 13 is informative about performance in a high-longevity scenario, in which the investor lives to age 95. Under fixed spending, IF-25% has the lowest probability of failure, at 20.1%, vs. 27.7% for WF-25% and 30.1% for WF-50%. The income-focused strategy also has the highest average, highest 10th percentile, and highest median spending. Under flexible spending, average and median spending are highest for WF-50%, while variability of income (both within and across simulated lifetimes) is lower for IF-25%. IF-25% has the highest 10th percentile of income.

EXHIBIT 13

Baseline Results for Fixed and Flexible Spending

WF-50% = 50% equity landing point and 5-year nominal bonds; WF-25% = 25% equity landing point and 5-year nominal bonds; IF-25% = 25% equity landing point and LDI portfolio. For a given simulation, average lifetime income is obtained by summing retirement spending in each year and dividing by realized longevity. The standard deviation of annual changes in spending and the difference between initial and minimal spending are both taken over the lifetime for a given simulation. For all quantities, the average, standard deviation, and percentiles are taken across 100,000 simulated histories. Initial assets at retirement are based on 40 annual contributions of $12,500 during the accumulation phase. The number of payments used to compute both fixed and flexible spending is based on the investor's conditional life expectancy times a 1.5 mortality buffer. For fixed spending, a constant amount in real terms is determined at age 65 based on the present value of equal annual payments starting immediately. For flexible spending, the investor updates her calculations annually, based on her current account balance and conditional life expectancy, which evolves with age. All numbers are inflation-adjusted.

PANEL A: STOCHASTIC MORTALITY

	Fixed Spending			Flexible Spending		
	WF-50%	WF-25%	IF-25%	WF-50%	WF-25%	IF-25%
Average lifetime income						
Average	57,007	53,665	55,860	66,178	57,001	59,756
Standard deviation	49,273	41,089	42,403	60,363	44,078	46,148
10th percentile	19,069	20,640	21,741	20,726	21,656	22,795
50th percentile	43,191	42,465	44,343	49,068	45,011	47,275
90th percentile	108,173	97,854	101,807	127,739	104,345	109,311
Avg. SD of annual changes	–	–	–	7,261	3,941	3,411
Avg. min. - init.	–	–	–	–14,379	–11,873	–10,590
% run out	13.11%	10.80%	8.51%	–	–	–
Avg. bequest	985,197	655,501	699,023	665,899	549,204	580,096

PANEL B: 30 DECUMULATION PERIODS

	Fixed Spending			Flexible Spending		
	WF-50%	WF-25%	IF-25%	WF-50%	WF-25%	IF-25%
Average lifetime income						
Average	54,701	52,295	55,124	67,231	55,942	58,847
Standard deviation	47,618	40,037	41,964	63,089	43,540	45,750
10th percentile	18,128	20,139	21,432	20,353	21,145	22,290
50th percentile	41,337	41,411	43,766	49,350	44,138	46,453
90th percentile	103,691	95,239	100,327	130,828	102,476	107,779
Avg. SD of annual changes	–	–	–	7,994	4,365	4,135
Avg. min. - init.	–	–	–	–21,062	–22,193	–22,254
% run out	30.13%	27.68%	20.16%	–	–	–
Avg. bequest	801,678	333,635	348,531	221,006	162,262	169,925

Hypothetical performance is no guarantee of future results.

For illustrative purposes only. All simulations are based on a hypothetical probability distribution, not historical data. See Appendix A for details.

Overall, the results show that the income-focused glide path offers a better tradeoff between average income and risk. Compared to WF-50%, IF-25% generates similar income under fixed spending, despite having a lower standard deviation of lifetime income and a lower failure rate. Under flexible spending, median lifetime income is lower but comparable, with sharply lower values for the standard deviation of both lifetime income and annual changes in spending.

The average income for IF-25% is always higher than WF-25%, even though both allocations have similar standard deviations of lifetime income. In fact, under fixed spending, the failure rate of IF-25% is lower. Under flexible spending, the standard deviation of annual changes in spending is also lower for IF-25%. These different measures of risk point in the same direction: the income-focused strategy offers a more favorable tradeoff between risk and average income. This pattern is consistent with Twardowski and Lennon (2019), who find that an LDI portfolio generates higher retirement income than a conventional bond portfolio, with less uncertainty.

The results of Panel B suggest that high equity exposure may not be adequate for managing longevity risk, especially for investors who value smooth consumption over time. Additional equity exposure in WF-50% does increase average income under flexible spending, but the improvement comes with increased variability. Under fixed spending, WF-50% has the highest failure rate and the lowest median spending. A potentially more effective way to manage longevity risk is to focus on the spending side of the equation. For individuals who wish to leave a bequest behind, using a longer planning horizon, which results in a lower spending rate, is one option. Annuities offer another option, to which we turn next.

4.2 Annuities

EXHIBIT 14

Baseline Results for Annuitization (stochastic mortality)

WF-50% = 50% equity landing point and 5-year nominal bonds; WF-25% = 25% equity landing point and 5-year nominal bonds; IF-25% = 25% equity landing point and LDI portfolio. For a given simulation, average lifetime income is obtained by summing retirement spending in each year and dividing by realized longevity. The difference between initial and minimal spending are both taken over the lifetime for a given simulation. For all quantities, the average, standard deviation, and percentiles are taken across 100,000 simulated histories. Initial assets at retirement are based on 40 annual contributions of $12,500 during the accumulation phase. Annuities are priced with mortality probabilities derived from Social Security Administration mortality tables. Payments for nominal and real annuities are discounted using the 10-year nominal rate and 10-year rate on inflation-indexed bonds, respectively. All numbers are inflation-adjusted.

	Nominal Annuity			Real Annuity		
	WF-50%	WF-25%	IF-25%	WF-50%	WF-25%	IF-25%
Average lifetime income						
Average	81,701	76,034	78,834	78,834	74,773	77,398
Standard deviation	70,524	58,380	60,397	60,397	56,924	58,560
10th percentile	27,525	29,236	30,359	30,359	29,062	30,380
50th percentile	61,773	60,038	62,324	62,324	59,314	61,489
90th percentile	155,189	138,795	144,020	144,020	135,945	140,800
Avg. min. - init.	–29,961	–27,874	–28,961	0	0	0

Hypothetical performance is no guarantee of future results.

For illustrative purposes only. All simulations are based on a hypothetical probability distribution, not historical data. See Appendix A for details.

Exhibit 14 shows the income generated by annuitization under each asset allocation. The income depends on the investor's assets at age 65 and the prevailing cost of the annuity, which varies with interest rates (higher rates imply a lower cost). Average income is similar across asset allocations but less volatile for strategies with more bonds. This is unsurprising given the results of Exhibit 12, which show that initial assets are similar across allocations but less volatile for allocations with lower equity exposure.

Average income is close for both nominal and real annuities because nominal yields are based on rational expectations about future inflation under our simulation setup. Therefore, on average, both types of annuities result in approximately the same average income over retirement. Nominal annuities result in more variability *within* retirement since the purchasing power of each payment will vary with inflation. When inflation is positive, higher longevity results in a larger decrease since inflation compounds over a longer period. For instance, a 2% inflation rate results in a 40% loss of purchasing power over 25 years.

The results of Exhibit 14 can be compared to the fixed and flexible spending results of Panel A in Exhibit 13. Compared to fixed spending, real annuitization generates 40.9% more income on average for WF-50% and 38.6% more for IF-25%. Annuities also have a failure rate of zero, and real annuities have no income fluctuations during retirement (unlike flexible spending).[6] Mortality pooling explains the higher income under annuitization.

This higher income comes at a cost. The investor loses control of her assets, which eliminates the possibility of making withdrawals to meet one-time expenses. She also gives up an average bequest of $985,197 ($593,593 median) for WF-50% and $699,023 ($505,579 median) for IF-25%. Overall, annuities can be valuable for investors willing to forgo the flexibility of retaining the assets and the possibility of leaving a bequest in exchange for longevity protection.

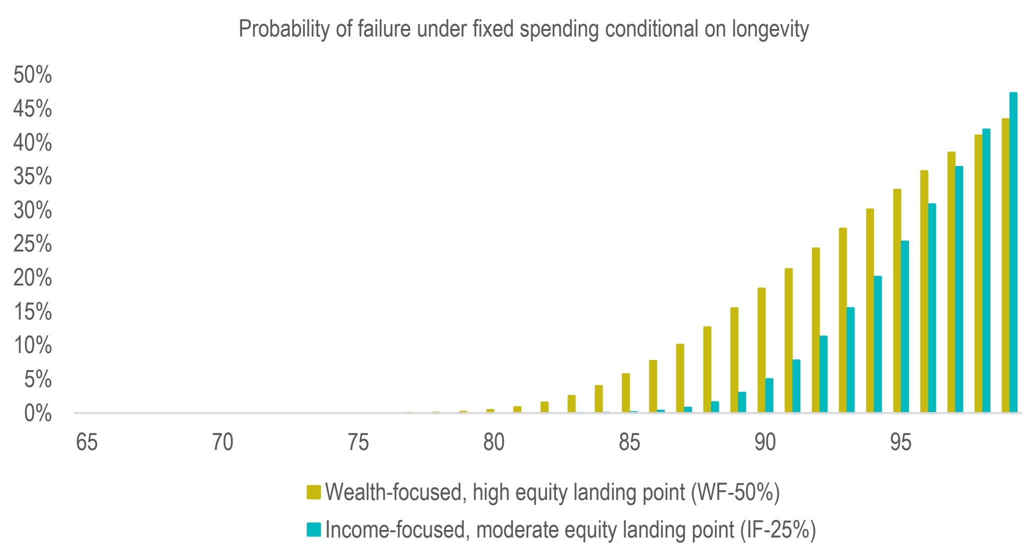
5. Evolution of Retirement Spending Over Time

In Section 4, we focused on the standard of living achieved by the investor in a given simulation, as measured by lifetime average retirement income. We now focus on retirement income at each age, from age 65 to 100. For brevity, we omit the wealth-focused allocation with a moderate equity landing point (WF-25%), as Exhibit 13 shows that it rarely achieves the best outcomes.

Exhibit 15 plots the cumulative probability of failure under fixed spending across all 100,000 simulations. The failure rate for the income-focused allocation (IF-25%) is virtually zero before age 85 and only 5% at 90. In contrast, the wealth-focused allocation with a moderate equity landing point (WF-50%) has a 6% failure rate at age 85, which increases to 18% by 90. When longevity is higher than the planning horizon assumed by the investor (that is, for ages 95 and above), both strategies have a high failure rate. WF-50%, which has a higher equity exposure, does not meaningfully improve outcomes when longevity is high.

EXHIBIT 15

Cumulative Probability of Failure Under Fixed Spending by Age



Hypothetical performance is no guarantee of future results.

For illustrative purposes only. All simulations are based on a hypothetical probability distribution, not historical data. See Appendix A for details.

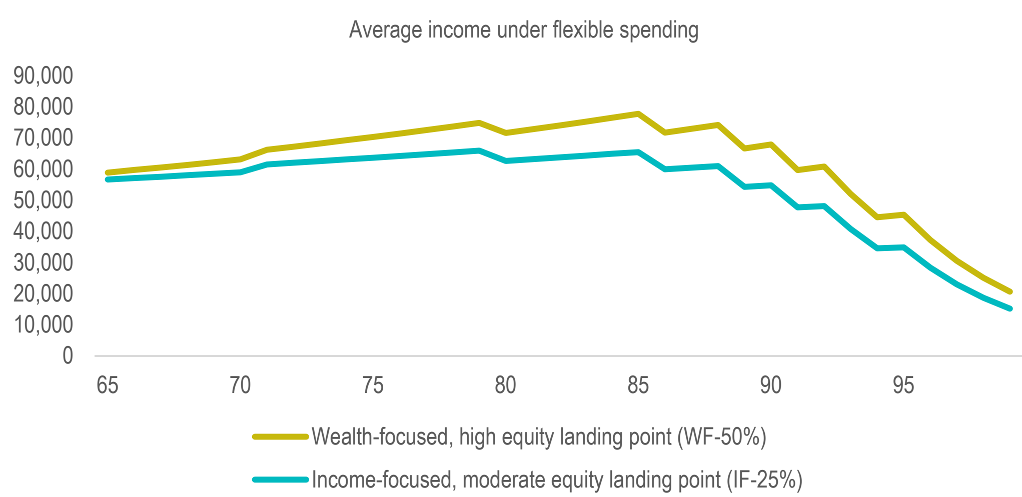
[6] Our simulations do not account for counterparty risk, an important consideration for real-world annuities.

Under flexible spending, the investor spends a percentage of her current balance, so her assets cannot be depleted and the failure rate is zero. However, retirement income can decrease substantially with age if the balance declines. **Exhibit 16** presents the average and 10th, 50th, and 90th percentiles for income at different ages under flexible spending. The patterns that hold for lifetime average retirement income in Exhibit 13 also hold for income at different ages: IF-25% has a higher income trajectory at the 10th percentile, similar for the median, and lower at the 90th percentile and on average. In all cases, the gaps narrow for ages closer to 100. The income-focused approach results in comparable retirement income not just over the course of the average retirement, but for income at different ages. The dispersion of outcomes by age is also significantly lower under the income-focused approach.

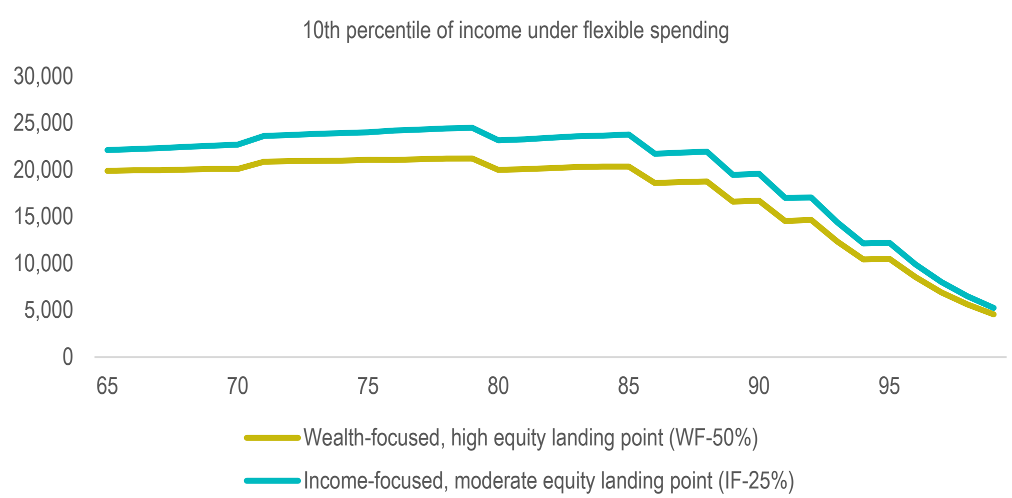
EXHIBIT 16

Average and Percentiles of Income Under Flexible Spending by Age

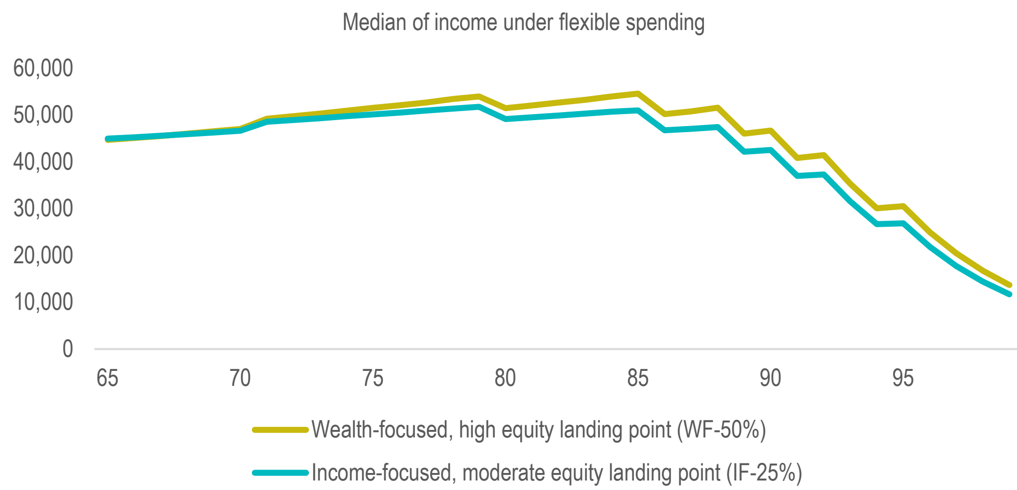
PANEL A: AVERAGE



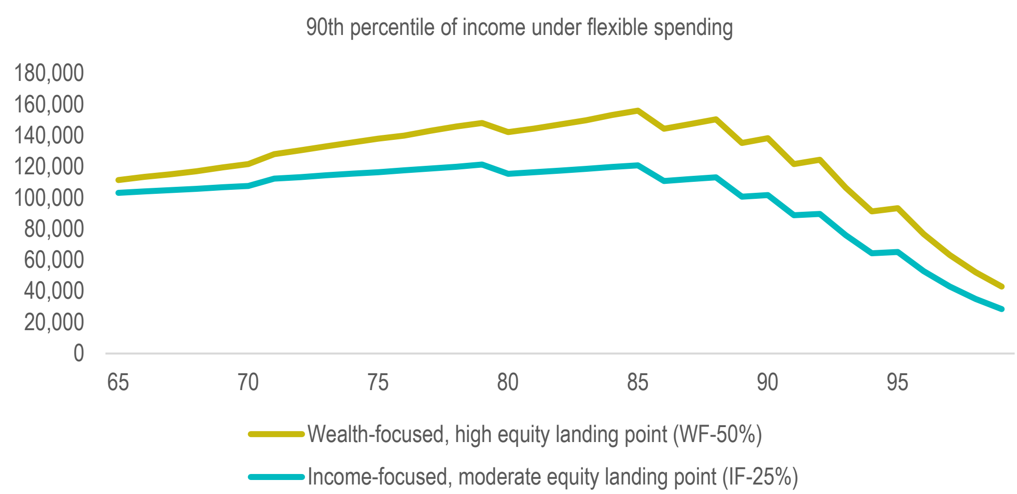
PANEL B: 10TH PERCENTILE



PANEL C: 50TH PERCENTILE



PANEL D: 90TH PERCENTILE



Hypothetical performance is no guarantee of future results.

For illustrative purposes only. All simulations are based on a hypothetical probability distribution, not historical data. See Appendix A for details.

6. Effects of Negative Shocks on Retirement Spending

We now consider the effects of different negative shocks (stock market drop, unexpected rise in inflation, and unexpected drop in interest rates) on retirement income. For all shocks, we consider the increase in the failure rate for fixed spending and the decrease in income for flexible spending, relative to the baseline results shown in Exhibit 15 and Exhibit 16.

For each shock, we focus on the 10,000 worst simulations out of the 100,000 based on the outcomes in the first five years of retirement. For stocks, this means that the average real return over these five years is –10.7%, compared to a full-sample average of 5%. For inflation, we consider the largest 10% unexpected inflation hikes: average inflation in the first five years of retirement is 3.7%, compared to 2% in all simulations. Finally, for real interest rates, we use the steepest 10% unexpected drops of the level factor, which are effectively a downward parallel shift of the curve (see Appendix for details): the five-year average of long-term interest rates is 1%, vs. 2% in the full sample. Although these shocks might appear moderate, they can have a substantial impact on retirement income. Larger increases in inflation or interest rate decreases would amplify the results reported below.

6.1 Stock Market Shock

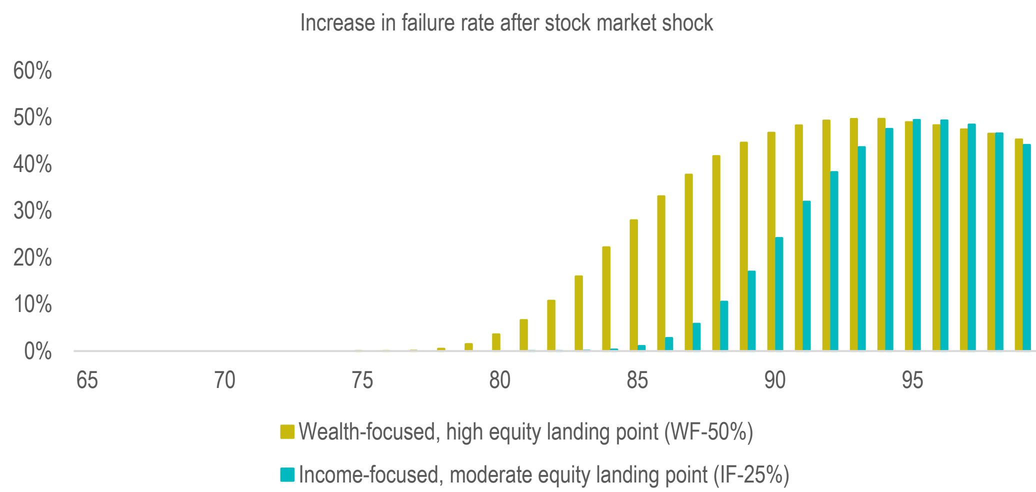
Exhibit 17 shows the effect of a stock market shock. For fixed spending, Panel A shows large increases in the failure rate for both the wealth-focused allocation with a high equity landing point (WF-50%) and the income-focused allocation (IF-25%). However, the increase starts much sooner for WF-50%. In the baseline results, the failure rate for WF-50% at age 85 is 6%. The increase conditional on a stock market shock is 28%, leading to a 34% total failure rate. By contrast, the unconditional failure rate at age 85 is less than 1% for IF-25%, and the failure rate conditional on a negative stock market shock is less than 2%. Failure rates become extremely high for both strategies if the investor lives past 90.

Panel B shows the percentage reduction in spending following the shock under flexible spending. Since stock returns are independent through time, a sequence of poor stock market returns has a permanent effect on the account balance. Investors reduce their spending by the same percentage as the decrease in their balance. The loss percentage is higher for WF-50% because of its higher exposure to equity: a stock market shock reduces lifetime income by about 34%, compared to 18% for IF-25%.

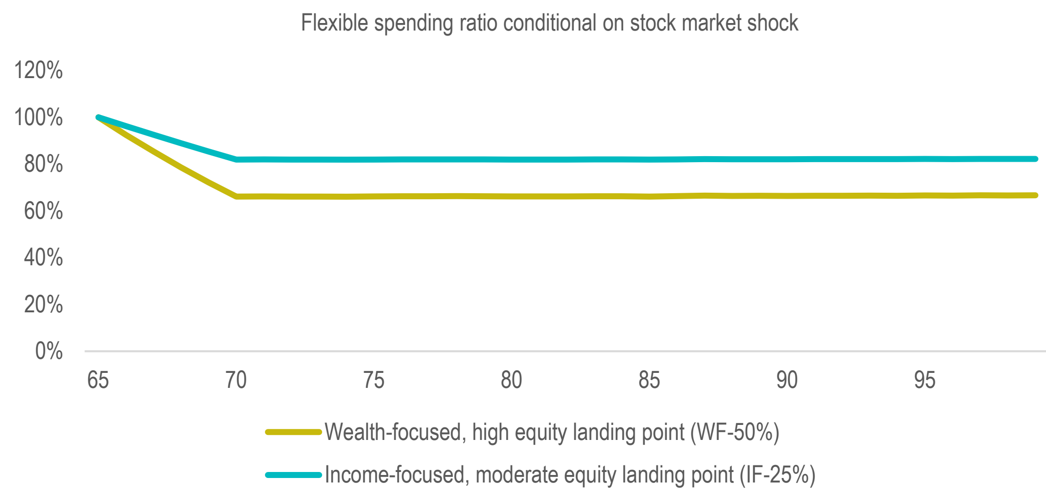
EXHIBIT 17

Effects of a Stock Market Shock on Retirement Income

PANEL A: INCREASE IN PROBABILITY OF FAILURE UNDER FIXED SPENDING



PANEL B: DECREASE IN SPENDING RELATIVE TO AVERAGE PATH UNDER FLEXIBLE SPENDING



Hypothetical performance is no guarantee of future results.

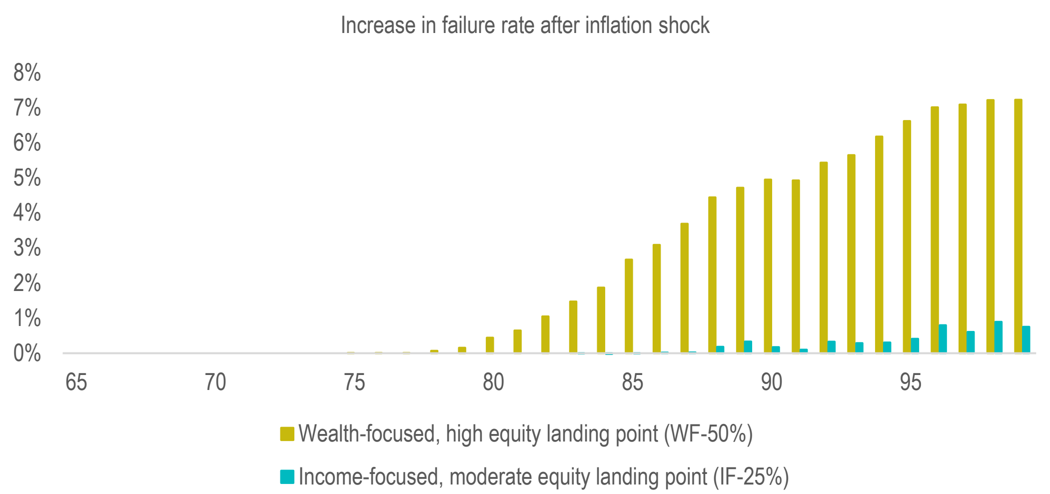
For illustrative purposes only. All simulations are based on a hypothetical probability distribution, not historical data. See Appendix A for details..

6.2 Inflation Shock

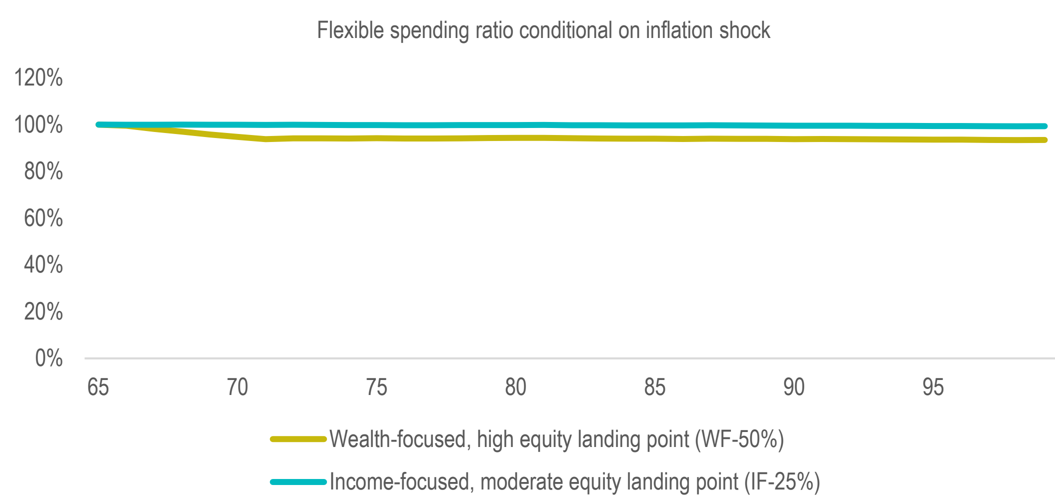
EXHIBIT 18

Effects of an Inflation Shock on Retirement Income

PANEL A: INCREASE IN PROBABILITY OF FAILURE UNDER FIXED SPENDING



PANEL B: DECREASE IN SPENDING RELATIVE TO AVERAGE PATH UNDER FLEXIBLE SPENDING



Hypothetical performance is no guarantee of future results.

For illustrative purposes only. All simulations are based on a hypothetical probability distribution, not historical data. See Appendix A for details.

As shown in **Exhibit 18**, an inflation shock has little effect on IF-25%. This is because the bond portfolio is indexed to inflation and because real stock returns are assumed to be independent of inflation. However, inflation has a negative impact on WF-50%. Panel A shows that, at age 90, the failure rate increases by approximately five percentage points, from 18% to 23%. At age 95, the failure rate increases from 33% to almost 40%. Panel B shows the situation for flexible spending. WF-50% incurs a permanent income loss of 6% on average.

In our simulation setup, shocks to inflation have persistent effects. Therefore, when the initial shock occurs, future inflation is expected to be high, and nominal yields increase to reflect this new information. This adjustment lowers nominal bond prices and creates a drag on returns. After the adjustment, expected returns on nominal bonds are equal to their average in baseline results. This is why the impact of an inflation shock early in retirement can be long-lasting: the lower-than-expected returns during the shock are not offset by higher-than-expected returns in the future.

6.3 Interest Rate Shock

Exhibit 19 shows the effect of a negative shock to real interest rates. The shock reduces yields at all maturities equally (parallel shift). Panel A shows that, under fixed spending, the failure rate increases for WF-50%. At age 90, the failure rate increases from 18% to 21%; at age 95, the failure rate increases from 33% to approximately 38%. The impact on IF-25% is minimal.

To understand the patterns in Panel A, note that two opposite effects occur: the initial drop in interest rates results in a capital gain on existing bond positions, but low yields depress future returns. For short-term instruments, such as the five-year notes used by the wealth-focused allocations, the latter effect dominates and reduces the probability that the bond portfolio will successfully fund future retirement spending. For an LDI portfolio such as the one used by the income-focused allocation, the two effects cancel each other.

The intuition for the LDI portfolio is easy to see in the case of a single-payment liability. Suppose the investor must make a payment of $100 in 10 years. If she holds a zero-coupon bond that pays $100 in 10 years, her ability to meet her liability is insensitive to changes in interest rates; the capital gain (or loss) exactly offsets changes in future returns. This is the goal of duration matching in the LDI portfolio. By contrast, if the investor were funding the same liability by investing in one-year notes that she must roll over at maturity, lower interest rates would reduce the ability of her portfolio to meet the liability.

The dynamics in Panel B, which shows flexible spending, are more subtle. An initial spending drop occurs, reaching 4% by the end of the shock for IF-25% and 14% for WF-50%. Under IF-25%, spending recovers to 100% of baseline within five years after the end of the shock, and slightly exceeds the baseline level in the long run. For WF-50%, spending also converges to 100% of baseline spending, although the recovery is slower. Five years after the end of the shock, spending is still 4% under baseline.

To understand the spending dynamics, first consider the effect of an interest rate shock on a portfolio of stocks. In our framework, stocks returns are independent from interest rates, so all the effects come from the spending side. Right after the shock, the investor cuts her spending since the projected cost of her retirement liability goes up. This spending cut increases her balance relative to baseline. Once the interest rate shock dissipates, the investor resumes her usual spending behavior but now has access to a larger balance. This produces a pattern of spending cuts in the short term, followed by an increase above baseline in the long run.

The impact of the interest rate shock on the fixed income sleeve depends on its composition. For IF-25%, because of duration matching, the increased value of the bond portfolio approximately offsets the decreased spending rate induced by low interest rates. With the 25% allocation to stocks, the impact of the shock on the entire portfolio ends up being a small, temporary decrease in spending, followed by a slight uptick in the long run.

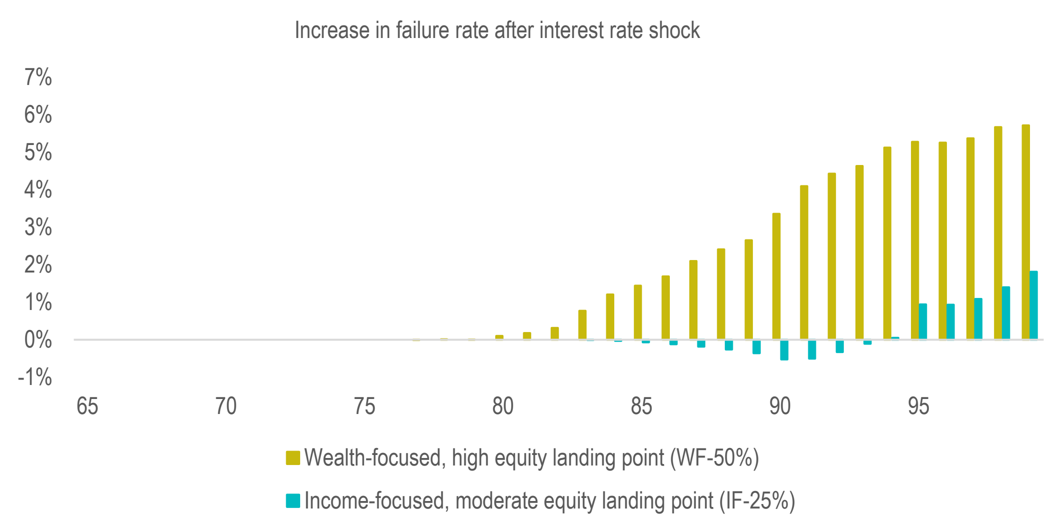
For WF-50%, the gain on short-term bonds does not fully offset the decrease in the spending rate. This results in a steeper spending drop in the short run. In the medium run, increased spending due to stocks approximately offsets the lower returns on bonds, and spending ends up close to its baseline value. The net effect of the shock is a temporary drop in spending that is not offset by future increases.

Overall, if the duration of the fixed income portfolio does not match the duration of the retirement liability, a drop in interest rates reduces the capacity of the portfolio to sustain retirement income. For fixed spending, this pattern manifests as a higher failure rate, as seen in Panel A of Exhibit 19. For flexible spending, in addition to lower return on short-term bonds, the decrease in interest rates causes the investor to spend more conservatively because she anticipates lower prospective returns on fixed income assets. For WF-50%, Panel B shows that these effects result in sharp, immediate spending cuts of about 15% early in retirement. By contrast, for both spending strategies, an income-focused approach reduces the impact of interest risk on spending.

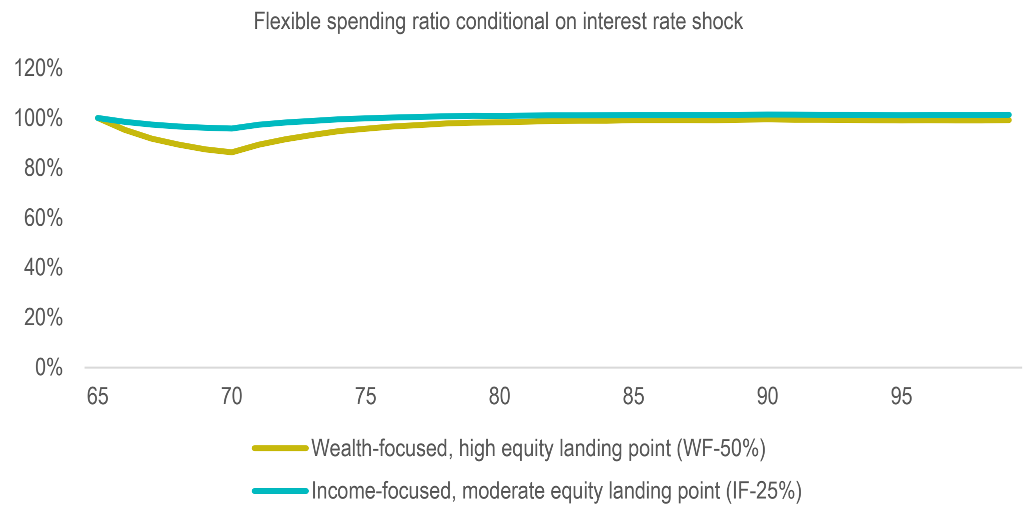
EXHIBIT 19

Effects of an Interest Rate Shock on Retirement Income

PANEL A: CHANGE IN PROBABILITY OF FAILURE UNDER FIXED SPENDING



PANEL B: DECREASE IN SPENDING RELATIVE TO AVERAGE PATH UNDER FLEXIBLE SPENDING



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For illustrative purposes only. All simulations are based on a hypothetical probability distribution, not historical data. See Appendix A for details.

7. Conclusion

Our results have a number of takeaways for retirement planning and glide path design. First, a glide path with a moderate allocation to equities at retirement can generate similar retirement income to a more aggressive allocation while significantly reducing the volatility of outcomes. Second, while long-maturity inflation-indexed bonds may be volatile in wealth terms, they can help manage inflation and interest rate risk, which ultimately reduces the volatility of retirement income. Third, an income-focused allocation combined with a well-thought-out spending plan can sustain retirement spending over several decades. When it comes to longevity risk, a high allocation to equities cannot substitute for proper risk management and retirement planning.

Appendix

A. Data-generating Process

The economic environment is defined by five variables. Each variable y_t is the sum of a long-term average μ and a perturbation ε_t that follows an AR(1) process, as shown below.

$$y_t = \mu + \epsilon_t$$
$$\epsilon_t = \rho\epsilon_{t-1} + z_t$$

The innovations z_t are independent draws from a normal distribution with mean zero. Given a calibrated value σ_y for the standard deviation of the observed variable y_t, the calibrated standard deviation of z_t equals $\sigma_z = \sigma_y \sqrt{1-\rho^2}$.

Exhibit A1 contains the calibrated values for the five processes. Values are chosen to generate trajectories consistent with US historical experience. The volatility of stock market returns is similar to its 1926–2020 estimated value, but the average stock market return is lower, consistent with Fama and French (2002).[7] Inflation dynamics reflect the post-1970s experience. Simulations therefore assume mean-reverting, moderate inflation.

EXHIBIT A1

Calibrated Values for the Data-generating Process

Variable (y_t)	Average (μ)	Standard Deviation of y_t	Persistence (ρ)
Stock market return (real)	5%	20%	0
Inflation	2%	1.5%	2/3
β_1 (level factor)	2%	1%	5/6
β_2 (slope factor)	–1%	1.12%	5/6
β_3 (curvature factor)	0%	0.96%	5/6

The real yield curve is modeled as a dynamic Nelson-Siegel process (DNS) with three independent factors, following Diebold and Li (2006). The yield on a bond of maturity τ at time t is given by

$$y_t(\tau) = (1)\cdot\beta_{1t} + \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right)\cdot\beta_{2t} + \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)\cdot\beta_{3t}.$$

The terms in parentheses are the factor loadings. Factor loadings are constant through time. The coefficient on β_1 is one for all maturities: a change in β_1 corresponds to a parallel shift of the entire curve; hence its interpretation as a level factor. The factor loading on β_2 decreases with maturity at a rate determined by λ. Finally, the factor loading on β_3 is a hump-shaped function: the value of λ determines at which maturity the peak is reached.

We use λ = 0.28. This choice ensures that, on average, yields increase quickly with maturity until the 10-year mark, then more gradually for long maturities. On average, the difference between the overnight rate (τ = 0) and the 10-year yield is 0.66%, compared to a difference of 1% between the overnight rate and long-maturity rates (rates when τ tends to infinity). This pattern is consistent with empirical data on term spreads.

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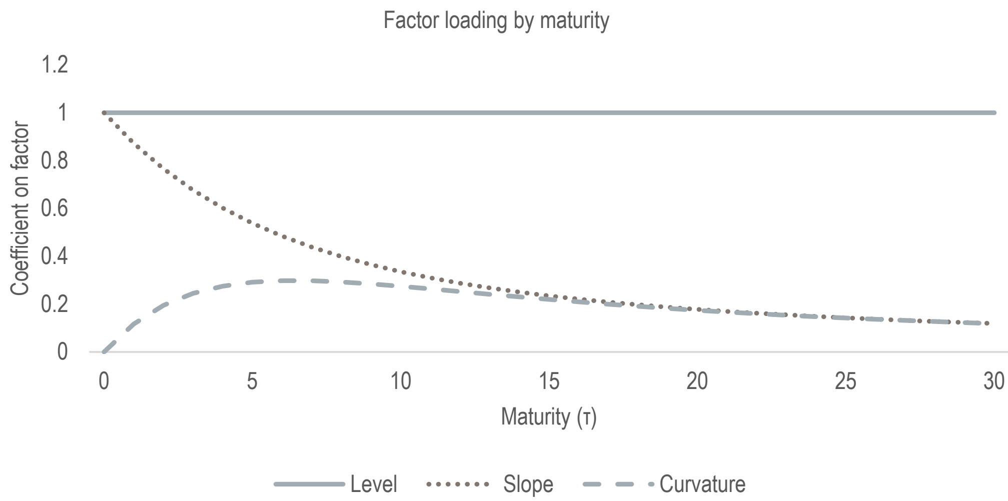
[7] Historical real returns are based on the Center for Research in Security Prices (CRSP) market portfolio and US CPI.

Exhibit A2 shows how the three factors affect different segments of the yield curve for our chosen calibration. A change in β_2 mostly affects short rates. The factor loading on β_2 decreases from 1 for the overnight ($\tau = 0$) rate to 0.12 for the 30-year yield; hence its interpretation as a slope factor. The effect of β_3 is most pronounced in the 5–7 years segment, in line with its usual interpretation as an intermediate-term or curvature factor.

EXHIBIT A2

Factor Loadings When λ = 0.28



The model generates a rising real yield curve in most simulations, with an average overnight yield of 1% and a long-term yield of 2%, but inverted and hump-shaped curves also occur. Factor volatilities are calibrated to yield a volatility of 1.5% for the overnight yield, which smoothly declines to 1% for long-term yields. The volatility and persistence of the three factors reflect historical data and stylized facts documented in the academic literature (e.g., Diebold and Li, 2006; Piazzezi, 2010). Real rates slowly revert to their mean following a shock.

Nominal yields are derived from real yields and inflation. For each maturity, expected inflation over the lifetime of the bond is added to the real yield to get the nominal yield. For simplicity, we assume an inflation risk premium of zero. In Appendix B, we show that adding a 50 basis point (bps) inflation risk premium to nominal yields does not materially affect the results. The calibrated model implies an average annual nominal return on five-year US Treasury notes of 1.8%, with a standard deviation of 4.3%, lower than the historical experience to reflect lower yields in recent decades.

Exhibit A3 shows the average yield curve. The average yield curve is obtained by evaluating the formula for $y_t(\tau)$ when β_1, β_2, and β_3 equal their expected values. Real/nominal yields rise from 1%/3% at $\tau = 0$ to 2%/4% for very long maturities. The 2% gap between the real and nominal curves corresponds to expected inflation. **Exhibit A4** shows the frequency of negative yields at different maturities for both real and nominal yields. Nominal and real rates at long maturities are rarely negative. The one-year real rate is negative about 20% of the time. **Exhibit A5** shows the frequency of different yield curve shapes. Yield curve shapes are defined by comparing the one-year, 10-year, and 30-year yields. A hump-shaped curve occurs when the 10-year yield is the highest of the three, and a U-shaped curve occurs when it is the lowest. Yields rise monotonically for both nominal and real yields about two-thirds of the time.

EXHIBIT A4

Average Yield Curve for λ = 0.28

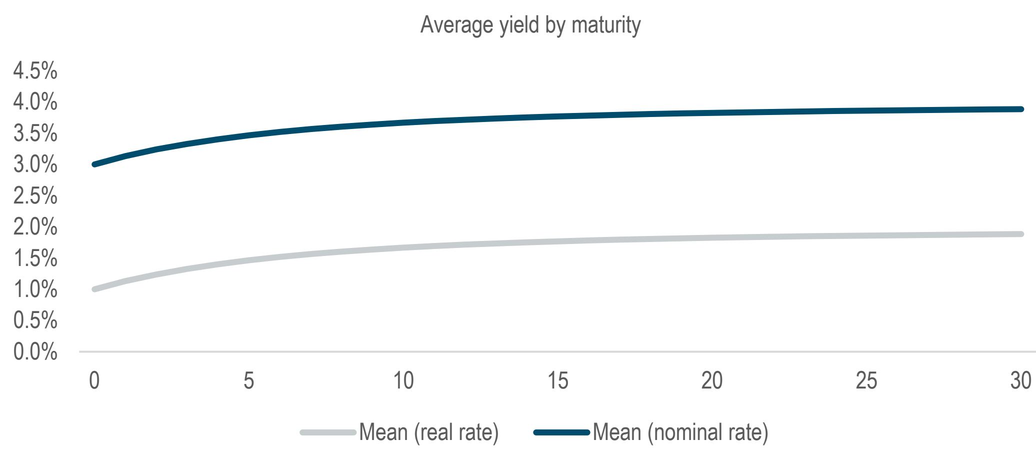


EXHIBIT A5

Percent of Periods with Negative Yields for Each Maturity

Maturity	% Negative (real)	% Negative (nominal)
1 year	21.0%	3.5%
10 year	6.6%	0.1%
30 year	3.2%	0.0%

EXHIBIT A6:

Percent of Periods with Different Yield Curve Shapes

Description	Real Yields	Nominal Yields
Monotonically increasing	70.1%	64.1%
Monotonically decreasing (inverted)	11.7%	19.0%
Hump-shaped	10.6%	7.5%
U-shaped	7.6%	9.5%

Mortality is based on data from the Social Security Administration.[8] Mortality probabilities are averaged across genders. These gender-neutral probabilities are then used to compute conditional life expectancies from age 65 to 119. The year 2020 is chosen as the baseline for mortality probabilities at age 65. Probabilities for age 66 are based on projected mortality in 2021, 67 on projected mortality in 2022, and so on. This approach accounts for projected mortality improvement when pricing annuities and projecting retirement spending.

[8] "Period Life Tables," Social Security Administration, 2020.

B. Robustness Checks

EXHIBIT B1

Aggregate Results When Stock Market Returns Are 6% on Average (+100 bps Compared to Baseline)

WF-50% = 50% equity landing point and 5-year nominal bonds; WF-25% = 25% equity landing point and 5-year nominal bonds; IF-25% = 25% equity landing point and LDI portfolio. For a given simulation, average lifetime income is obtained by summing retirement spending in each year and dividing by realized longevity. The standard deviation of annual changes in spending and the difference between initial and minimal spending are both taken over the lifetime for a given simulation. For all quantities, the average, standard deviation, and percentiles are taken across 25,000 simulated histories. Initial assets at retirement are based on 40 annual contributions of $12,500 during the accumulation phase. The number of payments used to compute both fixed and flexible spending is based on the investor's conditional life expectancy times a 1.5 mortality buffer. For fixed spending, a constant amount in real terms is determined at age 65 based on the present value of equal annual payments starting immediately. For flexible spending, the investor updates her calculations annually, based on her current account balance and conditional life expectancy, which evolves with age. All numbers are inflation-adjusted.

PANEL A: STOCHASTIC MORTALITY

	Fixed Spending			Flexible Spending		
	WF-50%	WF-25%	IF-25%	WF-50%	WF-25%	IF-25%
Average lifetime income						
Average	70,753	64,885	67,498	85,339	70,154	73,556
Standard deviation	62,836	51,175	52,497	79,512	55,361	57,763
10th percentile	22,861	24,067	25,337	25,882	25,694	27,193
50th percentile	53,013	50,850	52,857	62,442	54,862	57,518
90th percentile	136,800	121,052	124,788	166,449	129,897	136,638
Avg. SD of annual changes	–	–	–	9,406	4,876	4,240
Avg. min. - init.	–	–	–	–15,770	–13,192	–11,626
% run out	9.78%	8.74%	6.35%	–	–	–
Avg. bequest	1,424,635	856,099	914,355	873,659	680,960	719,996

PANEL B: 30 DECUMULATION PERIODS

	Fixed Spending			Flexible Spending		
	WF-50%	WF-25%	IF-25%	WF-50%	WF-25%	IF-25%
Average lifetime income						
Average	68,537	63,642	66,885	88,438	69,524	73,171
Standard deviation	60,831	50,051	51,872	84,184	55,109	57,782
10th percentile	22,130	23,752	25,103	26,019	25,393	26,806
50th percentile	51,406	49,806	52,375	64,217	54,248	57,031
90th percentile	132,801	118,767	123,920	174,001	129,104	136,098
Avg. SD of annual changes	–	–	–	10,548	5,457	5,197
Avg. min. - init.	–	–	–	–22,058	–24,207	–24,021
% run out	23.48%	21.82%	14.60%	–	–	–
Avg. bequest	1,294,967	491,818	523,234	312,758	209,441	219,523

Hypothetical performance is no guarantee of future results.

For illustrative purposes only. All simulations are based on a hypothetical probability distribution, not historical data. See Appendix A for details.

EXHIBIT B2

Aggregate Results When Long-term Yields Are 1% (–100 bps Compared to Baseline)

WF-50% = 50% equity landing point and 5-year nominal bonds; WF-25% = 25% equity landing point and 5-year nominal bonds; IF-25% = 25% equity landing point and LDI portfolio. For a given simulation, average lifetime income is obtained by summing retirement spending in each year and dividing by realized longevity. The standard deviation of annual changes in spending and the difference between initial and minimal spending are both taken over the lifetime for a given simulation. For all quantities, the average, standard deviation, and percentiles are taken across 25,000 simulated histories. Initial assets at retirement are based on 40 annual contributions of $12,500 during the accumulation phase. The number of payments used to compute both fixed and flexible spending is based on the investor's conditional life expectancy times a 1.5 mortality buffer. For fixed spending, a constant amount in real terms is determined at age 65 based on the present value of equal annual payments starting immediately. For flexible spending, the investor updates her calculations annually, based on her current account balance and conditional life expectancy, which evolves with age. All numbers are inflation-adjusted.

PANEL A: STOCHASTIC MORTALITY

	Fixed Spending			Flexible Spending		
	WF-50%	WF-25%	IF-25%	WF-50%	WF-25%	IF-25%
Average lifetime income						
Average	48,215	44,169	45,923	58,249	47,841	50,164
Standard deviation	41,319	33,428	34,304	52,357	36,333	38,002
10th percentile	16,182	17,030	17,891	18,069	18,155	19,077
50th percentile	36,438	34,912	36,351	43,201	37,750	39,594
90th percentile	92,101	80,570	83,453	113,778	87,575	91,892
Avg. SD of annual changes	–	–	–	6,514	3,415	2,934
Avg. min. - init.	–	–	–	–10,646	–8,972	–7,845
% run out	9.72%	8.47%	6.40%	–	–	–
Avg. bequest	971,055	606,282	646,102	642,816	501,067	528,921

PANEL B: 30 DECUMULATION PERIODS

	Fixed Spending			Flexible Spending		
	WF-50%	WF-25%	IF-25%	WF-50%	WF-25%	IF-25%
Average lifetime income						
Average	46,818	43,394	45,564	60,347	47,358	49,795
Standard deviation	40,069	32,630	34,002	55,674	36,191	37,958
10th percentile	15,667	16,796	17,789	18,025	17,894	18,858
50th percentile	35,380	34,288	36,089	44,404	37,369	39,335
90th percentile	89,356	79,191	82,601	118,407	86,962	91,049
Avg. SD of annual changes	–	–	–	7,330	3,818	3,609
Avg. min. - init.	–	–	–	–15,394	–16,974	–16,968
% run out	22.89%	21.10%	14.11%	–	–	–
Avg. bequest	799,730	306,347	322,617	213,067	142,534	148,974

Hypothetical performance is no guarantee of future results.

For illustrative purposes only. All simulations are based on a hypothetical probability distribution, not historical data. See Appendix A for details.

EXHIBIT B3

Aggregate Results When Inflation Risk Premium Is 0.5% (+50 bps Compared to Baseline)

WF-50% = 50% equity landing point and 5-year nominal bonds; WF-25% = 25% equity landing point and 5-year nominal bonds; IF-25% = 25% equity landing point and LDI portfolio. For a given simulation, average lifetime income is obtained by summing retirement spending in each year and dividing by realized longevity. The standard deviation of annual changes in spending and the difference between initial and minimal spending are both taken over the lifetime for a given simulation. For all quantities, the average, standard deviation, and percentiles are taken across 25,000 simulated histories. Initial assets at retirement are based on 40 annual contributions of $12,500 during the accumulation phase. The number of payments used to compute both fixed and flexible spending is based on the investor's conditional life expectancy times a 1.5 mortality buffer. For fixed spending, a constant amount in real terms is determined at age 65 based on the present value of equal annual payments starting immediately. For flexible spending, the investor updates her calculations annually, based on her current account balance and conditional life expectancy, which evolves with age. All numbers are inflation-adjusted.

PANEL A: STOCHASTIC MORTALITY

	Fixed Spending			Flexible Spending		
	WF-50%	WF-25%	IF-25%	WF-50%	WF-25%	IF-25%
Average lifetime income						
Average	58,864	56,019	56,102	69,829	61,306	60,063
Standard deviation	50,236	42,429	42,177	63,530	46,881	45,865
10th percentile	19,511	21,336	21,612	21,727	23,275	22,843
50th percentile	44,507	44,209	44,318	51,821	48,079	47,164
90th percentile	113,027	102,878	102,797	136,240	113,004	110,921
Avg. SD of annual changes	–	–	–	7,680	4,260	3,430
Avg. min. - init.	–	–	–	–13,674	–10,608	–10,427
% run out	11.18%	7.63%	8.38%	–	–	–
Avg. bequest	1,107,673	782,281	708,478	714,978	606,299	587,553

PANEL B: 30 DECUMULATION PERIODS

	Fixed Spending			Flexible Spending		
	WF-50%	WF-25%	IF-25%	WF-50%	WF-25%	IF-25%
Average lifetime income						
Average	56,801	55,101	55,365	71,584	61,045	59,112
Standard deviation	48,659	41,634	41,623	66,242	46,798	45,296
10th percentile	18,721	21,064	21,360	21,492	23,021	22,298
50th percentile	42,863	43,461	43,741	52,408	47,925	46,261
90th percentile	109,065	101,125	101,458	140,823	112,267	109,036
Avg. SD of annual changes	–	–	–	8,523	4,803	4,154
Avg. min. - init.	–	–	–	–19,877	–19,627	–22,293
% run out	26.53%	19.13%	19.90%	–	–	–
Avg. bequest	952,682	476,173	350,740	244,602	187,903	170,951

Hypothetical performance is no guarantee of future results.

For illustrative purposes only. All simulations are based on a hypothetical probability distribution, not historical data. See Appendix A for details.

With higher stock market returns (**Exhibit B1**), the average income and bequest are higher for all strategies, and failure rates under fixed spending are lower. For both spending rules, the 10th percentile of spending is highest for the income-focused allocation. Under fixed spending, IF-25% still generates comparable income to WF-50%, with a smaller standard deviation. However, the average bequest increases sharply (by more than $400,000 in both panels) for WF-50%. The high equity exposure during the decumulation period explains the gain. Under flexible spending, like in baseline results, the WF-50% generates a higher average income than IF-25% at the cost of more variability.

Lower bond yields (**Exhibit B2**) result in lower income for all strategies. The relative performance of the three glide paths is similar to baseline results for all spending strategies. Interestingly, failure rates under fixed spending and the maximal decrease ("Avg. min. – init") under flexible spending both fall. When rates are equal to their average under this specification, the initial spending rate for fixed spending is approximately 3.75%, compared to 4.25% in the baseline calibration. Lower rates reduce the returns of the bond sleeve but also induce investors to spend more conservatively; hence the observed pattern.

A 0.5% inflation risk premium (**Exhibit B3**) increases the expected return on five-year nominal bonds and raises the performance of the wealth-focused glide paths. The income-focused strategy is unaffected since it holds inflation-indexed bonds. The relative performance of WF-50% and IF-25% is similar to baseline results, while the performance of the WF-25% allocation improves substantially; it now generates comparable outcomes to IF-25%. The 0.5% inflation risk premium is large relative to both long-run inflation (2%) and the average yield on five-year inflation-indexed bonds (approximately 1.5%). The results suggest that, even when nominal bonds have a sizable inflation risk premium, LDI inflation-indexed bonds remain a valuable asset for income risk management.[9]

[9] Consider two one-period bonds held to maturity: one inflation-indexed, one nominal. Real yields are known at the beginning of the period, but realized inflation in that period is random. The real return on the inflation-indexed bond is known in advance and equal to the real yield. The nominal yield is equal to (Real yield) + (Inflation risk premium) + (Expected inflation), and the real return on the nominal bond equals (Real yield) + (Inflation risk premium) + (Expected inflation – Realized inflation). The last term is random for nominal bonds and separate from the inflation risk premium. By contrast, the return on inflation-indexed bonds does not vary with realized inflation. The same logic applies to multiperiod bonds.

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